HYDRAULIC MODEL OF A WATER COOLING SYSTEM IN A CHEMICAL PLANT

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Steady state hydraulic calculation has been described of an extensive pipeline network based on a new graph algorithm for setting up and decomposition of balance equations of the model. The parameters of the model are characteristics of individual sections of the network (pumps, pipes, and heat exchangers with armatures). In case of sections with controlled flow rate (variable characteristic), or sections with measured flow rate, the flow rates are direct inputs. The interactions of the network with the surroundings are accounted for by appropriate sources and sinks of individual nodes. The result of the calculation is the knowlege of all flow rates and pressure losses in the network. Automatic generation of the model equations utilizes an efficient (vector) fixing of the network topology and predominantly logical, not numerical operations based on the graph theory. The calculation proper utilizes a modification of the model by the method of linearization of characteristics, while the properties of the modified set of equations permit further decrease of the requirements on the computer. The described approach is suitable for the solution of practical problems even on lower category personal computers. The calculations are illustrated on an example of a simple network with uncontrolled and controlled flow rates of cooling water while one of the sections of the network is also a gravitational return flow of the cooling water.

A characteristic feature of chemical and petrochemical plants is usually considerable production of low potential heat with no further technological use that has to be removed. The most frequently used coolant for removal of this heat is cooling water. Apparatuses using cooling water (coolers, condensers) may number in tens or even hundreds. Transportation of cooling water necessitates frequently a very complex system of pipelines with pumps, exchangers and other armatures referred to as the pipeline network. Consumption of electricity to power the centrifugal pumps for cooling water may constitute a sizeable part of plant's electricity budget. Considerable operating and capital costs of the pipeline transform, through the price of the cooling water, also into the operating costs of the cooling apparatuses. The costs of cooling may be reduced primarily by rational technology (increased thermodynamic efficiency), by optimization of the operation of individual coolers and, last but not least, by optimization of the transport system for the cooling water.

A condition for optimum design, or operation of a pipeline is the hydraulic calculation resulting in the knowledge of the flow rates and pressure losses in all parts of the network and hence also in individual pumps and exchangers (specification of operating conditions of individual apparatuses).

Chemical engineering literature has devoted so far relatively little attention to the problem of design and analysis of pipeline networks (in contrast to other flow problems, e.g. in mixing equipment), although networks of various complexity constitute part of the majority of chemical technologies. Fundamental review of the calculation of the pipeline networks has been presented by e.g. Mah and Schacham[1]. The hydraulics of a pipeline network at steady state may be described in a manner analogous to the description of a DC circuit (Kirchhoff laws) with the difference that the elements of the pipeline network are nonlinear. The solution is thus always iterative by a number of available numerical techniques. With increasing complexity of the pipeline network, however, the decisive role is that of the problem formulation, i.e. setting up the model equations corresponding to the network of certain intrinsic structure (topology) and further the method of numerical solution of the resulting set of equations.

In our paper the hydraulic calculation of a complex cooling water pipeline network has been based on our own method of automated set up and decomposition of the model balance equations. This graph method starts from an efficient vector representation of the structure of a directed graph of the network using predominantly logical operations to set up the balance equations of the fundamental cutsets of the graph. The basis for further processing is a compactly formulated model due to Smith[2], which has been expanded, for our purposes, by the option of fixing certain flow rates within the network (zero flow rates for shut off sections, adjusted values for exchangers or measured values in the network). Using an example, a possibility is shown of fixing (implicitly) the pressure in more than one node of the network.

THEORETICAL

TOPOLOGY OF THE SYSTEM

The structure, or the topology, of pipeline network may be conveniently expressed in the form of a directed graph. In the following we shall introduce a few principal notions of the theory of graphs. A more detailed analysis may be found in the specialized literature[3]. A directed graph consists of a set of nodes (or vertices) and a set of directed edges (or streams) interconnecting individual nodes. In case of a pipeline network each edge represents a transport line of the cooling water, most of them a straight pipe of circular cross section. The orientation of the streams is conveniently defined in accord with the direction of the flow of liquid. In principle, however, both directions (defined and real) need not be identical. Each stream of a directed graph is incident with two nodes (initial and terminal). Each node can be incident with several streams. The number of these streams constitutes the degree of the node.

From the standpoint of the balance the nodes of the graph of the network function as mixers or separators. In a real system the node is, for instance, a branching point of the pipeline or a storage tank of water.

On neglecting the orientation of the edges one obtains an undirected graph. Since in the following we shall be making use of the properties of undirected graphs let us present some useful notions. Connection of two nodes by a continuous series of edges and nodes, in which each edge and node appears at most once, will be termed a simple path. The length of the path is given by the number of edges constituting the path. A simple path, for which the first and the last node coincide, will be termed a circuit. A graph whose two arbitrary points may be interconnected by a simple path will be termed connected graph. A connected graph containing no circuit will be termed a tree. A subgraph of a connected graph, which contains all nodes of the graph (partial graph), but only as much and such edges as not to make a circuit, is termed a spanning tree. The edges of the graph forming the spanning tree are called branches, the remaining are called chords. The partial graph, containing all chords is called cotree. The circuit formed by the branches of the spanning tree and just one chord is called a fundamental circuit. The edge cut between the nodes of a connected graph is such a set of edges, which, when left out, leaves just a disconnected graph (retaining one edge of the cut leaves the graph connected). If the cut contains just one branch of the spanning tree, it is called the fundamental cutset. Edge costed graph is given rise to by assigning each of its edges a real number (cost). Maximum spanning tree of the edge costed graph is the one having maximum sum of costed branches of all possible spanning trees. A set of edges and nodes possessing in an undirected graph the properties of a circuit is called, in the directed graph, the directed circuit. Contingent other terms from the theory of graphs will be explained directly in the text.

A pipeline network may form in principle a closed balance system. In case of industrial water cooling systems, however, we are dealing with open systems interconnected by mass streams with the surroundings. These, so-called, external streams are realized by transport lines which are not part of the pipeline network (e.g. evaporation into the atmosphere). Corresponding flow rates are then accounted for in the balance equations by means of external inputs (sources) and outputs (sinks). A directed graph of an open network may be transformed to a closed balance scheme by connecting the node of the surroundings.

The Principal Equations of the Model

Let us consider a hydraulic system schematized by a directed graph with V vertices and E directed edges, satisfying the following simplifying assumptions: *1*) the system is at steady state (constant flow rates and accumulation), *2*) the fluid is incompressible and flows isothermally, *3*) change of pressure takes place exclusively in streams, *4*) no violation of the suction head occurs in the network due to the breakage of a liquid column, 5) losses of liquid through leaks are negligible.

The mass balance of the system can then be expressed by V-1 independent equations for the nodes

$$\sum_{j=1}^{E} A_{ij} m_j = -m_i^* , \quad i = 1, 2, \ldots, V - 1 , \tag{1}$$

where A_{ij}, m_j, and m_i^* are elements of the reduced incidence matrix of the nodes **A**, vector of (internal) mass flow rates in the network $\boldsymbol{m}$, and the vector of source mass flow rates of the nodes $\boldsymbol{m}^*$, respectively. The elements of the incidence matrix **A** of dimensions $(V - 1) E$ are following:

$A_{ij} = 0$ the stream j is not incident with the node i,
$A_{ij} = 1$ the node i is the terminal node of the stream j,
$A_{ij} = -1$ the node i is the initial node of the stream j.

The elements of $\boldsymbol{m}$ and $\boldsymbol{m}^*$ are positive provided that the real direction of the flow is identical with the orientation of the stream of the graph, or, respectively, we are dealing with a source of mass in the node (feed from the surroundings of the graph). In case of a closed system, of course, the vector $\boldsymbol{m}^*$ is zero vector.

From the conditions of the model it follows that every node is characterized by a single pressure datum, i.e. the sum of pressure changes of liquid along an arbitrary circuit of the graph must equal zero. For $E - (V - 1)$ fundamental circuits of the graph one can write the following set of independent equations

$$\sum_{j=1}^{E} B_{kj} h_j = 0 , \quad k = 1, 2, \ldots, E - (V - 1) , \tag{2}$$

where B_{kj} and h_j are elements of the incidence matrix of fundamental circuits **B**, and vector of pressure losses by dissipation of mechanical energy $\boldsymbol{h}$, respectively. The elements of the incidence matrix of dimensions $(E - V + 1) E$ are given by the coefficients

$B_{kj} = 0$ the stream j is not part of the circuit k,
$B_{kj} = 1$ the stream j is part of the circuit k and oriented positively,
$B_{kj} = -1$ the stream j is part of the circuit k with opposite direction

(orientation of the fundamental circuit is chosen in accord with the orientation of the present chord).

The dependence of the elements of the vector $\boldsymbol{h}$ on the flow rate may be approximated by an empirical function

$$h_j = \alpha_j + \beta_j m_j + \gamma_j m_j^2 , \quad j = 1, 2, \ldots E , \tag{3}$$

where α_j, β_j, and γ_j are functions of the geometry of the sections of the pipeline network (pumps, pipes, armatures), physical properties and the flow rate of liquid, speed of revolution of the pump, etc. (For details see the example of the calculation).

The aim of the modelling of the system is to find values of the flow rates and pressure losses for all sections of the pipeline network. The flow rates $\boldsymbol{m}^*$ will be always taken to be a compulsory part of the problem statement.

Modification of the Model

The set of $V - 1$ independent balance equations can be made use of to express the same number of independent mass flow rates summarized in the vector $\boldsymbol{m}_T$. The characteristics (3) may then serve to calculate corresponding pressure losses $\boldsymbol{h}_T$. Additional $E - (V - 1)$ values of pressure losses and corresponding flow rates may be expressed from the set of equations (2) and (3) (vectors $\boldsymbol{m}_C$ and $\boldsymbol{h}_C$), or some (possibly all) of these flow rates may be directly given (fixed) as inputs (vectors $\boldsymbol{m}_{CG}$ and $\boldsymbol{h}_{CG}$).

The starting equations of the model (1) and (2) can then be rewritten as follows (decomposition)

$$\mathbf{A}_T \boldsymbol{m}_T + \mathbf{A}_C \boldsymbol{m}_C + \mathbf{A}_{CG} \boldsymbol{m}_{CG} = -\boldsymbol{m}^* \tag{4a}$$

$$\mathbf{B}_T \boldsymbol{h}_T + \mathbf{B}_C \boldsymbol{h}_C + \mathbf{B}_{CG} \boldsymbol{h}_{CG} = \boldsymbol{0}\,. \tag{4b}$$

It can be proved that the submatrix $\mathbf{A}_T$ of dimensions $(V - 1) \times (V - 1)$ is regular and represents the incidence matrix of the spanning tree of the graph[1,3]. The elements of the vector $\boldsymbol{m}_T$ thus correspond to the flow rates in the branches of the spanning tree; the remaining flow rates $\boldsymbol{m}_C$ and $\boldsymbol{m}_{CG}$ thus necessarily are chords.

Upon introducing now the so-called fundamental cutset matrix[1,3]

$$\mathbf{C} = \mathbf{A}_T^{-1}[\mathbf{A}_T, \mathbf{A}_C, \mathbf{A}_{CG}] \tag{5}$$

and upon considering the orthogonality of the matrices[3] $\mathbf{B}$ and $\mathbf{C}$

$$\mathbf{B}^T\mathbf{C} = \mathbf{C}\mathbf{B}^T = \mathbf{0}\,, \tag{6}$$

the set of Eqs (4) can be further arranged to

$$\mathbf{U}_T \boldsymbol{m}_T + \mathbf{C}_C \boldsymbol{m}_C + \mathbf{C}_{CG} \boldsymbol{m}_{CG} = -\mathbf{A}_T^{-1} \boldsymbol{m}^* \tag{7a}$$

$$\begin{bmatrix} -\mathbf{C}_C^T \\ -\mathbf{C}_{CG} \end{bmatrix} \boldsymbol{h}_T + \begin{bmatrix} \mathbf{U}_C \\ \mathbf{O} \end{bmatrix} \boldsymbol{h}_C + \begin{bmatrix} \mathbf{O} \\ \mathbf{U}_{CG} \end{bmatrix} \boldsymbol{h}_{CG} = \mathbf{0} \tag{7b}$$

which no longer contains the incidence matrix $\mathbf{B}$.

Upon replacing further the nonlinear characteristic (*3*) by its linearized form[2]

$$h_j = \alpha_j + R_{jj} m_j \,, \quad j = 1, 2, \dots E \,, \tag{8a}$$

where R_{jj} is an element of the diagonal matrix defined as follows

$$R_{ij} = \begin{cases} \beta_j + \gamma_j m_j = r_j \,; \quad i = j \\ 0; \; i \neq j \end{cases} \quad i, j = 1, 2 \dots E \tag{8b,c}$$

the set of E equations (*7*) can be finally arranged to give

$$\mathbf{\Delta} \boldsymbol{m}_C = \boldsymbol{\delta} \,, \tag{9a}$$

where the symmetric square matrix of the set $\mathbf{\Delta}$ of dimensions at most $(E - V + 1) \times (E - V + 1)$ and the vector of right hand sides $\boldsymbol{\delta}$ are given by

$$\mathbf{\Delta} = \mathbf{C}_C^T \mathbf{R}_T \mathbf{C}_C + \mathbf{R}_C \tag{9b}$$

and

$$\boldsymbol{\delta} = \mathbf{C}_C^T \mathbf{R}_T (-\mathbf{A}_T^{-1} \boldsymbol{m}^* - \mathbf{C}_{CG} \boldsymbol{m}_{CG}) + \mathbf{C}_C^T \boldsymbol{\alpha}_T - \boldsymbol{\alpha}_C \,. \tag{9c}$$

After solving the set of linear equations (*9*) for the unknowns $\boldsymbol{m}_C$ one calculate in a sequential manner the flow rates $\boldsymbol{m}_T$

$$\boldsymbol{m}_T = -\mathbf{C}_C \boldsymbol{m}_C - \mathbf{C}_{CG} \boldsymbol{m}_{CG} - \mathbf{A}_T^{-1} \boldsymbol{m}^* \,. \tag{10}$$

The loss of pressure by dissipation of mechanical energy of the streams (chords) for the fixed flow rate (vector $\boldsymbol{m}_{CG}$) can then be found as

$$\boldsymbol{h}_{CG} = \mathbf{C}_{CG} (\boldsymbol{\alpha}_T + \mathbf{R}_T \boldsymbol{m}_T) \,. \tag{11}$$

In case that no flow rate in the network has to be fixed the above approach reduces to the model published by Smith[2]. With the maximum number of flow rates in the network fixed, i.e. for all chords of the graph, the above model represents a just determined balance problem the solution of which is not affected by hydrodynamic properties of the network.

THE ALGORITHM OF DECOMPOSITION

The topology of the graph is set up most efficiently by a list of directed edges, giving for each edge its initial and terminal node[1]. Of principal importance for decomposition of the set of balance equations of the model is finding the spanning tree of the graph. As optimal, from the standpoint of the speed and stability of the calculation, appears the spanning tree corresponding to the set of fundamental circuits of minimum total length[4]. Search for these properties in the general case necessitates examination of the properties of all possible spanning trees of the graph (NP-complete problem[4]). In case of solution of practical problems, however, one can certainly do with an approximately optimum spanning tree found by means of some of the heuristic procedures. In our case we shall do with a simple and fast algorithm resting on setting up the maximum spanning tree of the edge costed graph[5]. The costing of edges here will be carried out in such a way that each edge is assigned a number indicating the sum of the degrees of both nodes with which it is incident. At the same time, those edges whose flow rates are taken to be part of the problem statement are discriminated, as far as their position in the constructed tree is concerned, by assigning low, e.g. zero, cost. Finding the spanning tree is followed by opposite procedure during which the spanning tree is gradually reduced to form a balance of fundamental cutsets through the balance of separated nodes. The result are the submatrices $\mathbf{C}_C$, $\mathbf{C}_{CG}$ and the vector $\mathbf{A}_T^{-1}\boldsymbol{m}^*$.

The applied graph algorithm may be briefly characterized as follows:

1) The edges of the graph are costed in a suitable way. The edge with maximum cost is found and one of its nodes opens the list of nodes of the tree. The list of branches is left empty for the moment.

2) The edge of the graph with maximum cost incident with just a single node of the tree is found. The edge is written into the list of branches and the list of nodes of the tree is expanded by its second node. The procedure is repeated until the edge of requested properties is found.

3) The found tree is the spanning tree of the graph. The edges of the graph that do not appear in the list of branches are listed in the list of chords. The list of edges, which are neither branches nor chords, is in this stage (spanning tree) empty.

4) The node is found incident with just a single branch of the tree (the last item of the current list of nodes of the tree or branches). The balance equation for this node is set up in such a way that the flow rate in the branch has a positive sign and the source term appears on the right hand side (substituting elements $+1$, -1, and 0 into the corresponding row of the fundamental cutset matrix, or the external input (output) flow rate value into the vector of right hand sides).

5) From the incident edges of the balanced node is selected the one that is neither chord nor branch of the tree (cancelled branch of the starting spanning tree). Its corresponding flow rate in the given balance equation is then eliminated by addition or subtraction of the earlier formulated balance equation. The procedure is repeated until the appropriate edge is found.

6) The result is the balance equation of the fundamental cutset in which, apart from the source term, appear only the flow rates in the chords and a single branch of the spanning tree. The list of nodes of the tree is reduced by the balanced node and the same is done also with the list of branches. The given branch is then written into the list of cancelled branches.

7) If the list of the branches of the tree is still not empty the routine returns to point *4*. In the opposite case the setting up and decomposition of the set of balance equations of the directed graph is finished.

Realization of the Calculation

Having set up and decomposed the mass balance equations presented by the graph algorithm one can proceed to the numerical solution of the model. With minor modifications one can use the method by Smith[2]:

1) Fixing flow rates $\mathbf{m}^*$ and $\mathbf{m}_{CG}$.

2) Initial guess of the flow rates $\mathbf{m}'_T$ and $\mathbf{m}'_C$ (preferably small but non zero values, not necessarily satisfying equation of continuity).

3) Calculation of the diagonal elements $\mathbf{R}_T$, $\mathbf{R}_C$ according to Eqs (*8a,b*).

4) Correction of the flow rates in the chords $\mathbf{m}_C$ by solving the set (*9*).

5) Correction of the flow rates in branches $\mathbf{m}_T$ by calculation from Eq. (*10*).

6) Judging the distance of the obtained solution $\mathbf{m}_T$ and $\mathbf{m}_C$ from the input data of the given approximation (marked by apostrophe)

$$M = \frac{\sum_{(j)} |(m_J)_j - (m_J)'_j|}{\sum_{(j)} |(m_J)'_j|}\,, \quad J = T, C\,. \tag{12a}$$

In case of acceptably low value of M see point *8*.

7) Calculation of the damping factor

$$N = (1/2)\exp(-M) \tag{12b}$$

and the combination of input data of the given approximation (marked by apostrophe) with the result of the calculation (damping of the oscillation of the solution)

$$N(\mathbf{m}_J)' + (1 - N)\,\mathbf{m}_J \rightarrow (\mathbf{m}_J)'\,, \quad J = T, C\,. \tag{12c}$$

Repeating of the routine from point *3*.

8) Calculation of values of losses of mechanical energy according to Eq. (*8*) or (*11*). For input reference value of pressure in one node, pressures in the remaining nodes are calculated. Termination of the calculation.

Example of Application

Description of System

In order to illustrate the described method let us use an example of a simple water cooling system sketched in Fig. 1*a*. This hydraulic system has a corresponding directed graph of **6** nodes and 10 streams shown in Fig. 1*b*. Description corresponding to Fig. 1 is given in Tables I and II. The pipeline network transports cooling water from a storage tank (node **1**) into two heat exchangers (streams *3* and *4*). At the same time this system serves as an emergency source of fire-extinguishing and technological water. The flow rate in the former of the two exchangers is controlled to a constant value (stream *3*) while the flow rate in the latter (stream *4*) is not currently corrected and adjusts thus to current pressure conditions prevailing in the network.

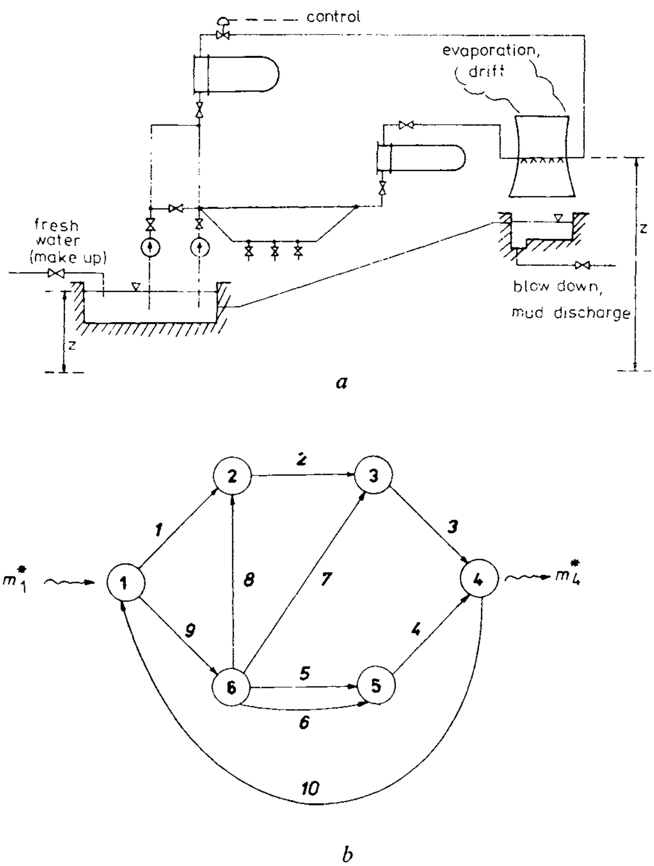

Fig. 1

Cooling system. *a* Technological scheme, *b* oriented graph (denotation see in the text)

The water warmed in exchangers is fed to a wet cooling tower to be cooled to the initial temperature. The removal of heat from the cooling water in the tower takes place predominantly due to its evaporation into the atmosphere[6]. Additional losses of water from the system occur by windage and drift or by sludge. The sum of these losses (flow rate m_4^*) represents the sink of the given node marked in Fig. 1*b* by a wavyline. The cooled water returns back by gravity into the storage tank (stream *10*, node **1**). Together with the returned cooling water the strorage tank enters fresh water compensating the losses (the source term of the balance node m_1^*, in Fig. 1*b* again marked by a wavyline). The cooling water is pumped by a pair of centrifugal pumps (streams *1* and *9*) while, at present demand on cooling, one of the pumps is idled (stream *1*). The described hydraulic system is thus opened via two nodes

TABLE I
Description of the graph of the network: streams

Stream	Initial node	Terminal node	Object of graph	Cost
1	1	2	pipe and pump (idled)	0
2	2	3	pipe	4
3	3	4	controlled flow exchanger	0
4	5	4	uncontrolled flow exchanger	5
5	6	5	pipe	8
6	6	5	emergency outlet pipe	8
7	6	3	pipe	7
8	6	2	pipe	7
9	1	6	pipe and pump	7
10	4	1	pipe	4

TABLE II
Description of the graph of the network: nodes

Node	Object	Degree of node
1	Cooling water storage tank, source	2
2	Pipe node	2
3	Pipe node	2
4	Cooling water and storage tank, sink	2
5	Pipe node	3
6	Pipe node	5

into the atmosphere (nodes **1** and **4**). The gravitational transport of the returned cooling water from node **4** to node **1** is possible thanks to the higher elevation of the node **4** (difference between the level of the inlet of the warmed water into the cooling tower and the water level in the storage tank).

Generation of the Balance Equations

Upon assigning the edges their appropriate costs (in Table I specifically the sum of the number of degrees of both nodes, while eliminating the edges with fixed flow rate) one finds branches of maximum spanning tree of the graph in the following sequence: ***5***, ***7***, ***8***, ***9***, and ***4***. The corresponding sequence of the adjoining nodes is **5**, **6**, **3**, **2**, **1**, and **4**. The found spanning tree is shown in Fig. 2. Now we are able to specify individual subvectors of the vector of mass flow rates as follows:

$$\boldsymbol{m}_\mathrm{T} = (m_5, m_7, m_8, m_9, m_4)\,, \quad \boldsymbol{m}_\mathrm{C} = (m_2, m_6, m_{10})\,, \tag{13a,b}$$

$$\boldsymbol{m}_\mathrm{CG} = (m_1, m_3)\,. \tag{13c}$$

In setting up the fundamental cutsets we shall make use (in the opposite sequence) of the sequence of the edges, originated in the process of forming the spanning tree of the graph. The resulting cutsets are shown in Fig. 3. By the first edge cut, encompassing the branch ***4*** and the chords ***3*** and ***10***, we shall separate and balance the node **4**, obtaining thus, while respecting the source term, the following

$$(0\ 0\ 0\ 0\ 1 \,|\, 0\ 0\ -1 \,|\, 0\ 1)\ \boldsymbol{m} = m_4^*\,. \tag{14a}$$

Similarly we then proceed for the node **1**

$$(0\ 0\ 0\ 1\ 0 \,|\, 0\ 0\ -1 \,|\, 1\ 0)\ \boldsymbol{m} = m_1^*\,, \tag{14b}$$

node **2**

$$(0\ 0\ 1\ 0\ 0 \,|\, -1\ 0\ 0 \,|\, 1\ 0)\ \boldsymbol{m} = 0\,, \tag{14c}$$

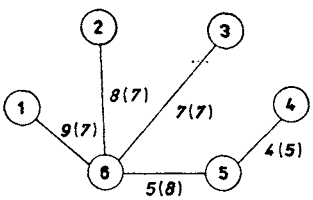

FIG. 2

Maximum spanning tree of the graph of the network (sum of costs in brackets: 34) and node **3**

$$(0\ 1\ 0\ 0\ 0\,|\,1\ 0\ 0\,|\,0\ -1)\quad \boldsymbol{m} = 0\,. \tag{14d}$$

In so far the performed cuts (in Fig. 3 designated as the first through the fourth cut) represented the separation and balancing of individual nodes of the starting graph. The last fundamental cutset (the fifth cut), encompassing the branch *5* and the chords *3*, *6*, *10*, however, splits the starting connected graph into two (connected) components with nodes **1**, **2**, **3**, and **6** and with nodes **4** and **5**. The sequence of the fundamental cutset is always taken such as to have in the separated component of the graph always a single so far unbalanced node (see the above outlined method). In this case there is still the node **6** left to be balanced while by the combination with the earlier found equations of all other parts of the given component of the graph one obtains the overall balance equation of the cutset (nodes **1**, **2**, **3**, and **6**):

$$(1\ 0\ 0\ 0\ 0\,|\,0\ 1\ 0\,|\,0\ 0)\quad \boldsymbol{m} = m_1^*\,. \tag{15}$$

Instead of the node **6** in our case we could balance node **5** (also the only so far unbalanced node of the second component of the graph), to obtain another acceptable form of Eq. (*15*).

The final result of the procedure of forming and balancing the fundamental cutsets is the identification of the matrix **C** and the vector $\mathbf{A}_T^{-1}\boldsymbol{m}^*$. Summarizing Eqs (*14*)

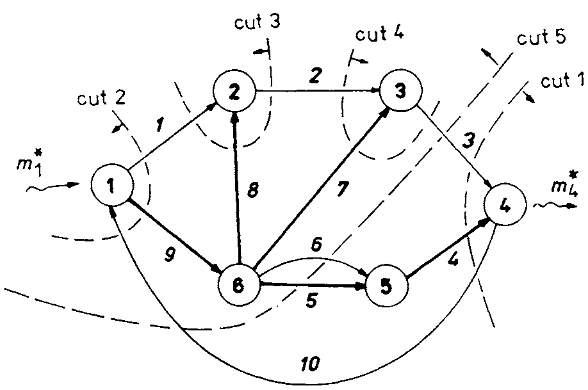

FIG. 3

Decomposition of the balance scheme by means of the fundamental cutsets (solid lines — branches, thin lines — chords, broken lines — fundamental cutsets) and (*15*) we thus obtain:

$$\mathbf{C} = \left[\begin{array}{ccccc|ccc|cc} 1&0&0&0&0 & 0&1&-1 & 0&1 \\ 0&1&0&0&0 & 1&0&0 & 0&-1 \\ 0&0&1&0&0 & -1&0&0 & 1&1 \\ 0&0&0&1&0 & 0&0&-1 & 1&0 \\ 0&0&0&0&1 & 0&0&-1 & 0&1 \end{array}\right], \quad -\mathbf{A}_T^{-1}\mathbf{m}^* = \begin{bmatrix} m_1^* \\ 0 \\ 0 \\ m_1^* \\ m_4^* \end{bmatrix} \qquad (16a,b)$$

without a priori knowledge of any other incidence matrix and without having to carry out numerical operations with its elements. Thus it is, in principle, not necessary to work with the whole matrix $\mathbf{C}$ (submatrix $\mathbf{C}_C$ is an identity matrix).

Fixing the Characteristics

Let us examine now the calculation of the elements $\boldsymbol{\alpha}_T$, $\boldsymbol{\alpha}_C$ and $\mathbf{R}_T$, $\mathbf{R}_C$, representing parameters of the linearized characteristic of individual streams, the flow rate of which is to be identified by the given model. Using the Darcy–Weisbach relation[7] and comparing it with Eq. (*3*) we obtain:

$$h_j = \zeta_j(1/2g)(4/\varrho\,\pi D_j^2)^2\,|m_j|\,m_j = \gamma_j m_j, \quad \alpha_j, \beta_j = 0\,, \qquad (17)$$

where ζ_j is the resistance coefficient. For a section of a straight pipe of length L and diameter D (the most common element of the pipeline network) we can rewrite this coefficient further as

$$\zeta_j = \lambda_j(L_j/D_j)\,, \qquad (18a)$$

while the friction coefficient in a tube, λ_j, is obtained from the White–Colebrook equation[7], modified (to speed up the calculation) to the following explicit form

$$\lambda_j = \lambda(\mathrm{Re}_j, D_j, \varepsilon_j) \qquad (18b)$$

e.g. by El-Abdalla[8]. For the case of so-called local resistances (e.g. measuring orifices, shut-off armatures, etc.) it is usually difficult to obtain the coefficient ζ_j with sufficient accuracy; reliable data can be obtained only by measurement of the arrangement in question. The results of measurements are, as a rule tabulated[7]. The magnitude of the local resistance is often characterized, analogously to Eq. (*18a*) for straight pipes, by the so-called equivalent length l_j. The magnitude of this length may depend on the direction of the flow of liquid through the given resistance.

Heat exchangers of the given cooling system represent generally a combination of both types of the resistances, i.e. the length as well as local ones. As an example, the tubular side of the exchanger (condenser) may thus be implemented into the graph as a sequence of several edges assigned for each passage, the resistances of the inlet and outlet from the bunch of tubes, bending of the stream between individual passages and the tubes of the exchanger[7]. With the knowledge of the geometry of the exchanger one can find by current means (tables, model equations) corresponding values of ζ_j, or λ_j. For more extensive cooling systems with numerous exchangers it is convenient, keeping the reduction of the extent of the problem in mind, to regard each exchanger as a single local resistance (length of the exchanger being usually small compared to the length of the pipe) for which the course of the overall characteristic (*3*) is evaluated before starting the calculation of the network. Similarly one can proceed, i.e. sum up the resistances, also for other more complex armatures of the network. In case of the intertubular space of the exchanger (usually single-pass with baffles) one uses in the modelling of the dissipation empirical relations encompassing geometrical as well as rheological properties of the exchanger. In order to avoid in our case description of the used exchangers we shall utilize here the simplest model of dissipation starting from the concept of equivalent length of the local resistance.

An unusual element in the example under consideration is the section with gravitational flow of the returned cooling water. Provided that this motion of the cooling water is realized at the expense of the potential energy, one can characterize the stream as follows

$$h_j = (\Delta z)_j = \alpha_j \,, \quad \beta_j, \gamma_j = 0 \,, \quad m_j \geqq 0 \,, \tag{19}$$

where Δz is the difference of the geodetic height between the initial and the terminal node of the given stream.

The characteristic of the centrifugal pump is used in the usual form of an empirical equation for the operating height

$$H_j = a_j + b_j m_j + c_j m_j^2 \,, \quad m_j \geqq 0 \tag{20a}$$

(monotonously decreasing function of the flow rate through the pump), while from comparison with the characteristic (*3*) we can derive

$$h_j = -H_j \,, \quad \alpha_j = -a_j \,, \quad \beta_j = -b_j \,, \quad \gamma_j = -c_j \,. \tag{20b}$$

The calculation proper. The input and output data of the model are given in Tables III and IV. In the calculation we have considered sections of the pipe of length $L = 100$ m and diameter $D = 0{\cdot}4$ m with absolute roughness $\varepsilon = 0{\cdot}004$ m. Only for the exchanger with uncontrolled flow rate of water (stream *4*) did we fix the overall length (the length of the pipe plus the equivalent length of the exchanger) at 1 000 m. The overall drop of the stream *10* was taken 20 m. Two flow rates of the cooling water are fixed in the network: zero flow rate of the stream *1* (idled pump) and the flow rate 250 kg/s of the stream *3* (currently maintained flow rate in the exchanger). The temperature of the cooling water was for all streams considered 20°C; in principle, however, this temperature may be fixed for each stream individually. There is one source of water defined in the network of strength 50 kg/s (node **1**) and equal sink (—50 kg/s in node **4**).

In the calculation we have executed altogether six iterative steps while the parameter, defined in Eq. (*12a*), decreased down to $M = 0{\cdot}000130$. The result of the calculation was the knowledge of the flow rates and pressure losses in all sections of the modelled network (Table III). In fixing the geodetic height z_i of individual nodes one can utilize the computed pressure losses for the calculation of pressure in individual nodes (quantity H_i in Table IV). For the reference pressure we took the pressure in the node **1** ($H_1 = 0$ m).

TABLE III

Parameters of the system: streams. $D_j = 0{\cdot}4$ m, $\varepsilon_j = 0{\cdot}004$ m, $t_j = 20$°C

Streams	Input data						Results[a]	
	$L_j + l_j$ m	z_j m	m_j kg/s	a_j m	$10^3 . b_j$ ms/kg	$10^6 . c_j$ ms²/kg²	m_j kg/s	h_j m
1	—	—	0	—	—	—	0	−42·46
2	100	—	0·001	—	—	—	103·53	−1·53
3	—	—	250		—	—	250	20·93
4	1 000	—	0·001	—	—	—	128·08	23·40
5	100		0·001	—	—	—	64·06	0·59
6	100	—	0·001	—	—	—	64·06	0·59
7	100	—	0·001	—		—	146·47	3·06
8	100	—	0·001	—			103·53	1·53
9	100	—	0·001	68·00	0·500	0·050	378·08	−43·99
10	—	20	0·001	—	—	—	328·08	20·00

[a] 6-th iteration: $M = 0{\cdot}000130$, $N = 0{\cdot}499935$.

TABLE IV

Parameters of the system: nodes

Parameter	Node 1	2	3	4	5	6
m_i^*, kg s^{-1}	50·0	—	—	—	—	−50·0
z_i, m	0	0	20	20	20	0
H_i, m	0	42·4	20·9	0	23·4	43·9

DISCUSSION

The application of the just described model to the dimensions of real cooling systems is practically solely a matter of computer technique. The computational procedure was therefore processed into the form of a computer program "SIT", written in BASIC 2 and debugged on a WANG 2 200 computer. The program makes use of the capability of the computer to work with alphanumeric strings[9]. In case that one works with nonnegative integers up to 255, these can be easily transformed into a symbolic form and the storage in the computer memory as elements of the string (alphanumeric variable) is then a great deal more efficient than in the usual case of a numeric variable. In such a way one can conveniently fix the topology of the system (altogether three vectors containing in the appropriate sequence numbers of streams, initial and terminal nodes). Of particular significance is this option in the case of the components of matrix of fundamental cutsets $\mathbf{C}_C$, $\mathbf{C}_{CG}$, whose elements may assume only three values (0, 1, and −1). A nonnegligible feature of the work with alphanumeric strings is the high speed of operation.

Additional savings of the internal computer memory in the program are achieved by respecting the specific properties of numerical matrices of the given model. Instead of the diagonal matrices $\mathbf{R}_T$ and $\mathbf{R}_C$ the program works with only the vectors of their diagonal elements $\boldsymbol{r}_T$, $\boldsymbol{r}_C$. The calculation of the symmetrical matrix $\boldsymbol{\Delta}$ is then programmed individually (not with the aid of standard operations with matrices) in such a way that we confine ourselves to elements of the triangular submatrix (shortening of the computation, saving of the memory). The set of linear equations (*9*) is then solved for the unknown vector $\boldsymbol{m}_C$ by decomposition according to Cholesky[10].

Respecting the above outlined principles permitted with using of about 55 kB of internal memory of the WANG 2 200 to solve the model of the pipeline network of cooling water consisting of 189 streams and 65 circuits (none of the internal flow rates was fixed). Seven iteration steps sufficed to achieve the requested accuracy of the solution (M = 0·000092).

Among the advantages of the given model is the simple (vectorial way of setting up the topology of the system serving, in a very rapid way without resorting to numerical operations, to formulate the balance equations of the model. The model further permits, depending on the need (e.g. cooling), or results of measurements in the network to fix certain flow rates within the network.

Possible changes of the statement of the problem from the standpoint of its topology and the flow rates within it, as well as the characteristics of the elements may thus be easily realized without having to modify the procedure itself. Apart from direct modifications by the operator there is the possibility of generating these modifications by the control program here. The given model may thus be used, for instance, within the framework of optimization calculations, necessitating solutions of a number of alternatives of the topology, characteristics and flow rates. The aim of the calculation is to find such a network ensuring cooling at minimum total costs. The hydraulic calculation then predicts the conditions of the work for each arrangement (flow rate, operating height of the pumps, pressure losses in control armatures, exchangers, etc.). The model may thus find application even in the control of the operation of the cooling system.

The model may, in principle, be used also for description of pipeline networks other than for the cooling water. The formulation of the balance equations has a general character and may thus be used regardless of the hydraulic properties of the streams (one-component balance). The model may contingently be expanded by enthalpy balances of nodes of the network, permitting, with the known temperatures of the external input fluxes and cooling power of individual coolers, calculation of temperatures of individual streams in the system. This option would be of significance particularly in case of more complex utilization of the cooling water in the system, for instance as a cascade of coolers.

The contribution of the just described method in particular is the efficient and rapid graph approach to generating the balance equations of the model of the system. Expansion of the model by the option of fixing some internal streams in the network facilitates modification of the conditions of operation of the network without having to change the topology of the system. A certain contribution is already the application of the method of hydraulic modelling to a water cooling system. A valuable finding is that the given model accepts also such transportation routes for liquid which are rather strange in conventional pipeline networks, for instance open troughs for gravitational transport of warmed water. In case of difficulties with convergence one can implement modifications in the method of setting up the spanning tree of the graph of the network. For instance, one can adopt a different method of costing the edges or expand the method by the optimization of the spanning tree[5]. Contingently, one can also modify the calculation of the damping factor. The cause for the failure of the calculation may, however, rest in the very subject as for certain conditions of the calculation the method need not have a real solution[1]. The calculation can be performed on personal computers of current category.

LIST OF SYMBOLS

A	reduced incidence matrix of nodes of graph
a	parameter of pump defined in Eq. (*20*), m
B	incidence matrix of fundamental circuits of graph
b	parameter of pump defined in Eq. (*20*), m s/kg
C	fundamental cutsets matrix of the graph
c	parameter of pump defined in Eq. (*20*), m s^2/kg^2
D	internal diameter of pipe, m
E	number of edges of graph

g	acceleration due to gravity, m/s^2
H	operating head of pump, m
$\boldsymbol{h}$	vector of pressure losses by dissipation of mechanical energy in streams, m
L	length of pipe, m
l	equivalent length, m
M	parameter defined in Eq. (*12a*)
$\boldsymbol{m}$	vector of mass flow rates of streams in graph, kg/s
$\boldsymbol{m}^*$	vector of sources in nodes of graph, kg/s
N	damping factor defined in Eq. (*12b*)
$\mathbf{0}$, $\boldsymbol{0}$	zero matrix, zero vector
$\boldsymbol{r}$	vector of diagonal elements $\mathbf{R}$, m s/kg
$\mathbf{R}$	diagonal matrix defined in Eq. (*8*), m s/kg
Re	Reynolds number
t	temperature, °C
$\mathbf{U}$	identity matrix
V	number of nodes (vertices) in graph
z	geodetic height, m
$\boldsymbol{\alpha}$	vector of parameters defined by Eq. (*3*), m
$\boldsymbol{\beta}$	vector of parameters defined by Eq. (*3*), m s/kg
$\boldsymbol{\gamma}$	vector of parameters defined by Eq. (*3*), m s^2/kg^2
Δ	matrix of set (*9*), m s/kg
$\boldsymbol{\delta}$	vector of right hand sides of set (*9*), m
ε	roughness of pipe, m
λ	coefficient of friction in straight pipe
ζ	coefficient of resistance
ϱ	density of liquid, kg/m^3

Subscripts

C	columns of matrix or elements of vector corresponding to chords with sought flowrate
CG	columns of matrix or elements of vector corresponding to chords with given (fixed) flow rate
i	general subscript (usually number of node)
j	general subscript (usually number of stream)
k	general subscript (usually number of circuit)
T	columns of matrix or elements of vector corresponding to branches of spanning tree of graph

Superscripts

′	input datum of approximation
T	transposed matrix
−1	inverse matrix

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